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Deformed matrix theories with $\mathcal{N} = 8$ and fivebranes in the PP wave background

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Abstract: M(atrix) theory is known to be mass-deformed in the pp-wave background and still retains all 16 dynamical supersymmetries. We consider generalization of such deformations on super Yang-Mills quantum mechanics (SYQM) with less supersymmetry. In particular this includes $\mathcal{N} = 8$ U(N) SYQM with a single adjoint and any number of fundamental hypermultiplets, which is a pp-wave deformation of DLCQ matrix theory of fivebranes. With $k \geq 1$ fivebranes, we show that a rich vacuum structure exists, with many continuous family of solutions that preserve all dynamical supersymmetries. The vacuum moduli space contains copies of CP^{k-1} of various sizes.

Keywords: Penrose limit and pp-wave background, M-Theory, M(atrix) Theories, D-branes.

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1. Introduction

It was discovered recently by Berenstein, Maldacena, and Nastase (BMN) [1] that $\mathcal{N}=16$ supersymmetric Yang-Mills quantum mechanics (SYQM) admits a massive deformation without sacrificing any of the supersymmetries. Instead, the supersymmetry algebra is deformed in such a way that hamiltonian no longer commutes with supercharges, implying that supersymmetry transformation itself is explicitly time dependent.

Usual SYQM with 16 supersymmetries [2] had played a crucial role in uncovering the 11-dimensional nature of M-theory [3]. The only known proposal for quantum formulation of M-theory, known as M(atrix) theory [4], is simply a large-N limit of U(N) SYQM with 16 supersymmetries, which can be thought of as large-N dynamics of D0 branes [5] or alternatively as a regularized dynamics of supermembranes [6]. In this regard, the deformed SYQM of BMN may be thought of as reformulation of M theory in a particular curved background which is a Penrose limit of either $AdS_7\times S^4$ or $AdS_4\times S^7$ [7]. Again, deformed SYQM may be considered as dynamics of D0 particles or as regularized dynamics of membranes in such a background [8]. The spacetime in question is often referred to as M-theory pp-wave background. Naturally one would like to ask what we can learn about M-theory by studying such mass-deformed version of M(atrix) theory. Some initial steps toward understanding quantum aspect of this deformed theory have been taken in ref. [9, 10].

One of more puzzling objects in M(atrix) theory is the fivebrane. Original proposal of M(atrix) theory apparently does not contain fivebranes as a solution, rather one must put them in by hand. For longitudinal fivebranes, this is achieved by introducing additional fields, namely fundamental hypermultiplets, in SYQM, thereby reducing dynamical supersymmetry by half to $\mathcal{N} = 8$ [11]. This quantum SYQM with 8 supersymmetries can also be used to study fivebranes themselves, and may be considered as an analog of M(atrix) theory for worldvolume dynamics of fivebranes, namely a Discrete-Lightcone-Quantization (DLCQ) description. In this note, we consider general $\mathcal{N} = 8$ SYQM and their mass deformation analogous to BMN's, thereby initiating the inquiry as to whether such deformed M(atrix) theory with fivebranes allows better understanding of fivebranes in particular.

Reducing supersymmetry to $\mathcal{N} = 8$ also involves reducing R-symmetries. Prior to mass deformation, $\mathcal{N} = 16$ SYQM are equipped with SO(9) R-symmetry coming from spatial rotations. Reduction to $\mathcal{N} = 8$ breaks up this to SU(2) $\times$ SO(5), where SO(5) is associated with directions orthogonal to fivebranes. In the deformed SYQM of BMN with $\mathcal{N} = 16$, however, SO(9) is already reduced to a smaller SO(3) $\times$ SO(6) R-symmetry by the four-form flux. Thus, depending on how we embed fivebranes, we will find different theories, not all of which may be supersymmetric.

In terms of fivebrane in the pp-wave background, this is because the background itself comes from $AdS_7 \times S^4$ (or $AdS_4 \times S^7$), a near-horizon limit of fivebranes, so not all orientations of the secondary fivebrane are equivalent. After some experiment with spinors, we discover that there is one particular subset of orientation that is amenable to reduction of supersymmetry and thus to introduction of additional supersymmetric fivebranes. Let us describe this choice in some detail. We start with $AdS_7 \times S^4$ such that $1 + 10$ directions are distributed as follows,

$$x^{0,1,2,3,4,8,9} \rightarrow AdS_7 \,, \tag{1.1}$$

$$x^{5,6,7,10} \rightarrow S^4 \,. \tag{1.2}$$

Perform the pp-wave limit of this background by taking $x^{\pm} = x^0 \pm x^{10}$. The BMN deformed M(atrix) theory then has SO(3)$\times$SO(6) R-symmetry where SO(6) rotates $x^{1,2,3,4,8,9}$ while SO(3) rotates $x^{5,6,7}$. Reduction of dynamical supersymmetry to $\mathcal{N} = 8$ is achieved in part by breaking up $x^{1,2,3,\dots,9}$ into five coordinates associated with vector multiplets and four coordinates associated with hypermultiplets. In this note, we take the following decomposition,

$$\begin{aligned} x^{1,2,3,4,5} &\rightarrow \text{vector}\,, \\ x^{6,7,8,9} &\rightarrow \text{hyper}\,. \end{aligned} \tag{1.3}$$

The resulting SYQM will have

$$\mathrm{SO}(4)_{1234} \times \mathrm{SO}(2)_{67-89}\,, \tag{1.4}$$

as the R-symmetry. SO(4) rotates directions 1,2,3,4, among themselves while SO(2) is a simultaneous and opposite rotation of planes 6,7 and 8,9.

This choice of decomposition seems special in the following sense. Recall that in the deformed SYQM, the supercharge does not commute with hamiltonian and the supermultiplet does not imply degeneracy. In particular, bosonic and fermionic degrees of freedom come with different excitation masses. Only after summing over all vacuum energies, one find cancellation among them giving a supersymmetric ground state of zero energy. The above decomposition into vector and hyper is unique up to irrelevant rotation, in that this cancellation of vacuum energy holds in each sector. This can be seen easily once we recall how the cancellation occurred in full $\mathcal{N}=16$ BMN case,

$$3\times\frac{|\mu|}{3}+6\times\frac{|\mu|}{6}-8\times\frac{|\mu|}{4}=0\,, \tag{1.5}$$

where 3 is from $x^{5,6,7}$, 6 is from $x^{1,2,3,4,8,9}$, and 8 is from 16 real fermions. With the above decomposition into vector and hyper then, this is rewritten as

$$\left(1\times\frac{|\mu|}{3}+4\times\frac{|\mu|}{6}-4\times\frac{|\mu|}{4}\right)+\left(2\times\frac{|\mu|}{3}+2\times\frac{|\mu|}{6}-4\times\frac{|\mu|}{4}\right)=0\,, \tag{1.6}$$

where each group gives zero separately. The first group represents vacuum energy from a vector multiplet, and the second represents vacuum energy from a hypermultiplet. In particular, this already suggests that we may truncate to vector part only or add additional hypermultiplets in the SYQM without destroying supersymmetry. This allows us to start with familair DLCQ theory of fivebranes and try to deform it in a manner similar to BMN. Whether there is other kind of deformations giving us fivebranes of different orientations remains uncertain and is beyond the scope of this paper.

In section 2, we will start with mass deformed $\mathcal{N}=16$ SYQM and rewrite it in $\mathcal{N}=8$ language. By either adding or subtracting hypermultiplet into the latter setup, we show that general $\mathcal{N}=8$ SYQM also admits massive deformation with all 8 supercharges remaining. In section 3, we outline how $\mathcal{N}=16$ superalgebra is truncated to $\mathcal{N}=8$ with emphasis on central charge. In section 4, we concentrate on the case with fundamental hypermultiplets which is DLCQ of fivebranes in pp-wave background, and isolate classical ground states that preserves all 8 dynamical supersymmetries. These solutions are analogous to the fuzzy sphere solutions of BMN which are essentially giant gravitons. Unlike the BMN case, however, we find that the solutions come in a continuous family typically parametrized by a CP^n vacuum moduli space. In section 4 we conclude with remarks on future direction of research.

2. Mass deformed $\mathcal{N}=8$ SYQM

2.1 10 dim Yang-Mills and deformed matrix theory in real representation

Here we review the mass deformed SYQM of BMN and set up notations and conventions. With signature $(-,++,\ldots,+)$ and $\bar\Psi^a=(\Psi^a)^T\Gamma^0$, we start with supersymmetric Yang-Mills theory in ten dimensions,

$$L=\mathrm{Tr}\left\{-\frac{1}{4}F^2-\frac{i}{2}\bar\Psi\Gamma^P D_P\Psi\right\}, \tag{2.1}$$

where A_P is hermitian and $D_P\Psi=\partial_P\Psi-i[A_P,\Psi]$.

The 10 dimensional Gamma matrices can be chosen to be real

$$\begin{aligned}\Gamma^0 &= 1 \otimes i\sigma_2\,, \\ \Gamma_I &= \bar{\gamma}_I \otimes \sigma_1 \qquad (I = 1, 2 \ldots, 9)\,, \\ \Gamma^{11} &= 1 \otimes \sigma_3\,,\end{aligned} \tag{2.2}$$

where $\bar{\gamma}_I$ is the symmetric real 16×16 gamma matrices of SO(9). The gaugino field is Majorana and is also chiral,

$$\Gamma_{11}\Psi = \Psi\,. \tag{2.3}$$

Note that $\Gamma^0\Gamma_I\Psi = \bar{\gamma}_I\Psi$. Because of this, we may as well regard $\bar{\gamma}_I$ as 32×32 matrices

$$\bar{\gamma}_I = \Gamma^0\Gamma_I \tag{2.4}$$

with the understanding that the spinor Ψ is restricted to be Majorana and chiral. This latter definition of $\bar{\gamma}_I$ naturally extends to complex representation of spinors we will later adopt.

Supersymmetric transformation is

$$\begin{aligned}\delta A_I &= -i\bar{\epsilon}\Gamma_I\Psi\,, \\ \delta\Psi &= \frac{1}{2}F_{IJ}\Gamma^{IJ}\epsilon\,.\end{aligned} \tag{2.5}$$

Let us dimensionally reduce this field theory to a quantum mechanics by allowing time-dependence only. With new notations, $X_I = A_I$ and A_0, the above lagrangian becomes [2]

$$L = \mathrm{Tr}\left\{\sum_I \frac{1}{2}(D_0X_I)^2 + \frac{1}{4}\sum_{IJ}[X_I, X_J]^2 + \frac{i}{2}\Psi^T D_0\Psi - \frac{1}{2}\Psi^T\bar{\gamma}_I[X_I, \Psi]\right\}. \tag{2.6}$$

The susy transformation of the quantum mechanics becomes

$$\begin{aligned}\delta A_0 &= i\Psi^T\epsilon\,, \\ \delta X_I &= i\Psi^T\bar{\gamma}_I\epsilon\,, \\ \delta\Psi &= D_0X_I\bar{\gamma}_I\epsilon - \frac{i}{2}[X_I, X_J]\bar{\gamma}_{IJ}\epsilon\,.\end{aligned} \tag{2.7}$$

BMN [1] deformed this quantum mechanics by adding the following set of terms, with one mass parameter μ,

$$\begin{aligned}\Delta L = &\frac{1}{2}\,\mathrm{Tr}\left(-\left(\frac{\mu}{3}\right)^2\sum_{a=5,6,7}(X_a)^2 - \left(\frac{\mu}{6}\right)^2\sum_{s=1,2,3,4,8.9}(X_s)^2\right) + \\ &+ \frac{1}{2}\,\mathrm{Tr}\left(\frac{2i\mu}{3}\epsilon_{abc}X_aX_bX_c - \frac{i\mu}{4}\Psi^T\bar{\gamma}_{567}\Psi\right),\end{aligned} \tag{2.8}$$

upon which the susy transformation get deformed to

$$\begin{aligned}\delta A_0 &= i\Psi^T\epsilon\,, \\ \delta X_I &= i\Psi^T\bar{\gamma}_I\epsilon\,, \\ \delta\Psi &= \left(D_0X_I\bar{\gamma}_I - \frac{i}{2}[X_I, X_J]\bar{\gamma}_{IJ} + \frac{\mu}{3}\sum_{a=5,6,7}X_a\bar{\gamma}_a\bar{\gamma}_{567} - \frac{\mu}{6}\sum_{s=1,2,3,4,8,9}X_s\bar{\gamma}_s\bar{\gamma}_{567}\right)\epsilon\,.\end{aligned} \tag{2.9}$$

The deformed lagrangian is invariant under this supersymmetry if and only if we force the following explicit time-dependence of the transformation parameter ϵ,

$$\epsilon(t) = e^{-\frac{\mu}{12}\bar{\gamma}_{567}t}\epsilon(0)\,, \tag{2.10}$$

which makes the superalgebra quite unconventional. In particular, the supercharges does not commute with hamiltonian, but raises or lower energy by $\mu/12$ unit. The notion of "supermultiplet" no longer implies degeneracy in this deformed superalgebra. Note that we chose SO(3) part of SO(9) to lie along direction $x^{5,6,7}$ for later convenience.

2.2 Complex representation and undeformed $\mathcal{N} = 8$ SYQM

In order to reduce to SYQM with $\mathcal{N} = 8$ supersymmetry and introduce other kind of supermultiplet, we break up the SO(9) spinor in terms of tensor product of SO(4) and SO(5) spinors. First introduce $e_\mu = (\boldsymbol{\sigma}, i)$ and $\bar{e}_\mu = (\boldsymbol{\sigma}, -i)$ and define the euclidean SO(5) gamma matrices[1]

$$\begin{aligned}\gamma_\mu &= \begin{pmatrix} 0 & e_\mu \\ \bar{e}_\mu & 0 \end{pmatrix}, \\ \gamma_5 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\end{aligned} \tag{2.11}$$

which implies

$$\begin{aligned}\gamma_1 &= \sigma_1 \otimes \sigma_1\,, \\ \gamma_2 &= \sigma_2 \otimes \sigma_1\,, \\ \gamma_3 &= \sigma_3 \otimes \sigma_1\,, \\ \gamma_4 &= -1 \otimes \sigma_2\,, \\ \gamma_5 &= 1 \otimes \sigma_3\,.\end{aligned} \tag{2.12}$$

Note that $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. With this, ten dimensional Dirac matrices can be written in a complex form as

$$\begin{aligned}\Gamma^0 &= 1 \otimes 1 \otimes i\sigma_2 \otimes 1 \otimes 1\,, \\ \Gamma_\mu &= \gamma_\mu \otimes \sigma_1 \otimes 1 \otimes 1\,, \qquad (\mu = 1, 2, \ldots, 5)\,, \\ \Gamma_{5+\mu} &= 1 \otimes 1 \otimes \sigma_3 \otimes \gamma_\mu\,, \qquad (\mu = 1, 2, 3, 4)\,, \\ \Gamma_{11} &= 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3\,,\end{aligned} \tag{2.13}$$

with 1 denoting the 2×2 identity matrix. As before, $\Gamma_{11} = \Gamma^0\Gamma_1 \cdots \Gamma_{10}$.

Since we abandoned real SO(9) matrices, we must impose Majorana condition. Defining complex conjugation by

$$\Psi_c = B\Psi^*\,, \tag{2.14}$$

with the matrix B such that

$$B\Gamma^I B^{-1} = (\Gamma^I)^*\,. \tag{2.15}$$

[1]To be distinguished from $\{\bar{\gamma}_I\} = \{\bar{\gamma}_s, \bar{\gamma}_a\}$

The Majorana condition is then $\Psi_c = \Psi$. ($C = B\Gamma^0$ and $C\Gamma^I C^{-1} = -(\Gamma^I)^T$ with $\Psi_c = C\bar{\Psi}^T$.) In our matrix convention

$$B = -\sigma_2 \otimes \sigma_3 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \,. \tag{2.16}$$

The above Yang-Mills lagrangian and supersymmetry work fine if we insist λ and ϵ are both chiral and Majorana. In this notation, one starts with a 32-component complex spinor, then the Majorana condition together with $\Gamma_{11}\Psi = \Psi$ reduces it to 16 real in effect.

We will decompose the spinor Ψ into two eight-component complex spinors,

$$\lambda^i_\alpha \,, \qquad \chi^i_\alpha \tag{2.17}$$

which are respectively chiral and anti-chiral under $\Gamma^0\Gamma^1\cdots\Gamma^5$. Similarly, supersymmetry parameter ϵ is split into ϵ_i and ζ_i. The 10-dimensional chirality condition then implies that λ_i and ϵ_i are of $(+,+)$ eigenvalues under the two σ_3 factors in the above form of Γ_{11} while χ_i and ζ_i are of $(-,-)$. Eventually we will keep the supersymmetries generated by ϵ_i's only. The Majorana condition becomes symplectic Majorana conditions

$$\lambda_i = -\tilde{B}(\sigma_2)_{ij}\lambda^*_j \,, \qquad \chi_i = +\tilde{B}(\sigma_2)_{ij}\chi^*_j \,, \tag{2.18}$$

where $\tilde{B} = \sigma_2 \otimes \sigma_3$ satisfies $\tilde{B}\gamma^\mu\tilde{B}^{-1} = \gamma^{\mu *}$ with $\mu = 1, 2, \ldots, 5$. Note that the indices $i, j = 1, 2$ belong to the fourth factor of spinor decomposition (2.13); σ_2 in the preceding equation is the fourth entry in the definition of B in eq. (2.16) above.

For fermions associated with hypermultiplet, the gauge representation could be either real or complex, so we may as well take one of χ_i's and discard the other. We will take $\chi = \chi_1$ as the fermion of the hypermultiplet. Similarly we introduce the two complex scalar belonging to the hypermultiplet.

$$\sum_{\mu=1}^{4} X_{\mu+5} e_\mu = \begin{pmatrix} \bar{y}_1 & y_2 \\ \bar{y}_2 & -y_1 \end{pmatrix} , \tag{2.19}$$

where $y_1 = X_8 - iX_9$ and $y_2 = X_6 - iX_7$.

With the above decomposition, the pure $\mathcal{N} = 8$ SYQM without mass deformation has

$$L_0 = \mathrm{Tr}\left(\frac{1}{2}\sum_{\mu=1}^{5}(D_0 X_\mu)^2 + \frac{1}{4}\sum_{\mu,\nu}^{5}[X_\mu, X_\nu]^2 + \frac{i}{2}\lambda^\dagger_i D_0 \lambda_i - \frac{1}{2}\lambda^\dagger_i \gamma_\mu [X_\mu, \lambda_i] + \frac{1}{2}\mathbf{D}^2 \right) , \tag{2.20}$$

which is invariant under

$$\begin{aligned} \delta A_0 &= i\lambda^\dagger_i \epsilon_i \,, \\ \delta X_\mu &= i\lambda^\dagger_i \gamma_\mu \epsilon_i \,, \\ \delta\lambda_i &= D_0 X_\mu \gamma_\mu \epsilon_i - \frac{i}{2}[X_\mu, X_\nu]\gamma_{\mu\nu}\epsilon_i + i\mathbf{D}\cdot\boldsymbol{\sigma}_{ij}\epsilon_j \,, \\ \delta\mathbf{D} &= -\epsilon^\dagger_i \boldsymbol{\sigma}_{ij} D_0 \lambda_j - i\epsilon^\dagger_i \boldsymbol{\sigma}_{ij}\gamma_\mu [X_\mu, \lambda_j] \,. \end{aligned} \tag{2.21}$$

Adding an adjoint hyper matter field is a matter of rewriting $\mathcal{N}=16$ undeformed SYQM in terms of the above decomposition

$$
\begin{aligned}
L_{adj} = \mathrm{Tr}\biggl(&\frac{1}{2}D_0\bar{y}_i D_0 y_i + \frac{1}{2}[X_\mu,\bar{y}_i][X_\mu,y_i] + \frac{1}{2}\mathbf{D}\cdot\boldsymbol{\sigma}_{ij}[\bar{y}_j,y_i] + \\
&+ i\chi^\dagger D_0\chi + \chi^\dagger\gamma_\mu[X_\mu,\chi] + \lambda_i^\dagger[\bar{y}_i,\chi] + \chi^\dagger[y_i,\lambda_i]\biggr)\,. \qquad (2.22)
\end{aligned}
$$

The combined action is invariant under the above susy transformation if we also transform,

$$
\begin{aligned}
\delta\bar{y}_i &= -2i\chi^\dagger\epsilon_i\,, \\
\delta\chi &= -D_0 y_i\epsilon_i - i\gamma_\mu[X_\mu,y_i]\epsilon_i\,. \qquad (2.23)
\end{aligned}
$$

Adding the fundamental matter field q,ψ in the $\bar{N}$ dimensional representation becomes

$$
\begin{aligned}
L_{\mathrm{fund}} = \mathrm{Tr}\biggl(&\frac{1}{2}D_0\bar{q}_i D_0 q_i - \frac{1}{2}X_\mu\bar{q}_i q_i X_\mu + \frac{1}{2}\mathbf{D}\cdot\boldsymbol{\sigma}_{ij}\bar{q}_j q_i + \\
&+ i\psi^\dagger D_0\psi - \psi^\dagger\gamma_\mu\psi X_\mu + \lambda_i^\dagger\bar{q}_i\psi + \psi^\dagger q_i\lambda_i\biggr)\,. \qquad (2.24)
\end{aligned}
$$

The susy transformation extends to

$$
\begin{aligned}
\delta\bar{q}_i &= -2i\psi^\dagger\epsilon_i\,, \\
\delta\psi &= -D_0 q_i\epsilon_i + iq_i X_\mu\gamma_\mu\epsilon_i\,. \qquad (2.25)
\end{aligned}
$$

2.3 Deformation of pure $\mathcal{N}=8$ SYQM

The above decomposition of spinors gives the relations

$$
\bar{\gamma}^{567} = \gamma^5\otimes\sigma_3\otimes i\sigma_3\otimes 1\,. \qquad (2.26)
$$

When acting on ϵ_i's and on λ_i's, the σ_3 in the middle takes $+1$ eigenvalue, while $i\sigma_3$ acts on $i=1,2$ indices. Thus we find that

$$
\epsilon_i(t) = \left(e^{-\frac{i\mu t}{12}\gamma_5\otimes\sigma_3}\right)_{ij}(\epsilon_0)_j\,. \qquad (2.27)
$$

with the matrix multiplication of γ^5 on the 4-component spinors $\epsilon_{1,2}$ understood.

Let us introduce one more notation. Since $\lambda_{1,2}$ are related by reality condition and so are $\epsilon_{1,2}$, define

$$
\begin{aligned}
\lambda_1 &= \lambda\,, \\
\epsilon_1 &= -i\epsilon\,, \qquad (2.28)
\end{aligned}
$$

from which it follows

$$
\begin{aligned}
\lambda_2 &= -i\tilde{B}\lambda^*\,, \\
\epsilon_2 &= \tilde{B}\epsilon^*\,, \qquad (2.29)
\end{aligned}
$$

and is consistent with the above time-dependence of ϵ_i's.

The vector multiplet lagrangian becomes

$$L_0 = \mathrm{Tr}\left(\sum_{\mu=1}^{5}\frac{1}{2}(D_0X_\mu)^2 + \sum_{\mu,\nu}^{5}\frac{1}{4}[X_\mu,X_\nu]^2 + i\lambda^\dagger D_0\lambda - \lambda^\dagger\gamma_\mu[X_\mu,\lambda] + \frac{1}{2}\mathbf{D}^2\right). \tag{2.30}$$

The additional piece of lagrangian that deforms a pure $\mathcal{N}=8$ SYQM is then

$$\Delta L_0 = \mathrm{Tr}\left(-\frac{1}{2}\left(\frac{\mu}{6}\right)^2\sum_{\mu=1}^{4}X_\mu^2 - \frac{1}{2}\left(\frac{\mu}{3}\right)^2X_5^2 + \frac{\mu}{4}\lambda^\dagger\gamma_5\lambda\right). \tag{2.31}$$

The lagrangian $L_0+\Delta L_0$ is invariant under the susy

$$\begin{aligned}
\delta A_0 &= (\lambda^\dagger\epsilon + \epsilon^\dagger\lambda)\,,\\
\delta X_\mu &= (\lambda^\dagger\gamma_\mu\epsilon + \epsilon^\dagger\gamma_\mu\lambda)\,,\\
\delta\lambda &= -iD_0X_\mu\gamma_\mu\epsilon - \sum_{\mu,\nu}^{5}\frac{1}{2}[X_\mu,X_\nu]\gamma_{\mu\nu}\epsilon - \frac{\mu}{6}\sum_{\mu=1}^{4}X_\mu\gamma_\mu\gamma_5\epsilon + \frac{\mu}{3}X_5\epsilon\,,
\end{aligned} \tag{2.32}$$

with

$$\epsilon(t) = e^{-\frac{i\mu}{12}\gamma_5 t}\epsilon_0\,. \tag{2.33}$$

This deformed pure $\mathcal{N}=8$ SYQM theory has 16 supersymmetry out of which eight is linearly realized, or dynamical, and eight is nonlinearly realized, or kinematical.

Note that the oscillator mass of the bosonic and fermionic degrees of freedom are again different from each other. Of five scalars, one is of mass $|\mu/3|$, namely X^5, while the remaining four are of mass $|\mu/6|$. As before, all four complex fermions are of mass $|\mu/4|$. The vacuum energy is then,

$$\dim(\mathrm{U}(N)) \times \left(\frac{|\mu|}{3} + 4\times\frac{|\mu|}{6} - 4\times\frac{|\mu|}{4}\right) = 0, \tag{2.34}$$

as promised. It has been shown recently that such vacuum energy is protected from correction by supersymmetry in $\mathcal{N}=16$ setting [10], which should also hold in $\mathcal{N}=8$ case.

2.4 BMN in complex notation: deformation of $\mathcal{N}=8$ SYQM with an adjoint hypermultiplet

The hypermultiplet part of the deformed lagrangian can be written

$$\Delta L_{adj} = \mathrm{Tr}\left(-\frac{1}{2}\left(\frac{\mu}{6}\right)^2\bar{y}_1y_1 - \frac{1}{2}\left(\frac{\mu}{3}\right)^2\bar{y}_2y_2 - \frac{1}{2}\mu X_5[\bar{y}_2,y_2] - \frac{\mu}{4}\chi^\dagger\gamma_5\chi\right). \tag{2.35}$$

We must recall that susy transformation of vector multiplet has an additional piece,

$$\delta\lambda = \cdots + iD^3\epsilon_1 + (iD^1+D^2)\epsilon_2\,, \tag{2.36}$$

which become nontrivial when hypermultiplets are present. In addition the hypermultiplet fields must transform as

$$\begin{aligned}
\delta\bar{y}_i &= -2i\chi^\dagger\epsilon_i(t)\,,\\
\delta\chi &= -D_0y_i\epsilon_i - i\sum_{\mu=1}^{5}[X_\mu,y_i]\gamma_\mu\epsilon_i + \frac{i\mu}{6}\gamma_5y_1\epsilon_1 + \frac{i\mu}{3}\gamma_5y_2\epsilon_2\,.
\end{aligned} \tag{2.37}$$

Recall that $\epsilon_1=-i\epsilon$ and $\epsilon_2=-i\tilde{B}\epsilon_1^*$.

This completes rewriting of BMN's deformed M(atrix) theory in terms of $\mathcal{N}=8$ supersymmetry. While obvious, it is worthwhile to point out that vacuum energy contribution from hypermultiplet degrees of freedom cancel among themselves. Bosons split into two real with mass $|\mu/3|$ and two real with $|\mu/6|$ while all four complex fermions are of mass $|\mu/4|$, so that

$$\dim(\mathrm{U}(N)) \times \left(2\times\frac{|\mu|}{3}+2\times\frac{|\mu|}{6}-4\times\frac{|\mu|}{4}\right)=0\,. \tag{2.38}$$

2.5 Deformation with fundamental hypermultiplet

Finally we are ready to add other hypermultiplets. To be specific, let us add fundamental hypermultiplets for U(N) while for other multiplets, this should carry over verbatim. Let q_i complex scalars and ψ complex spinor in representation $\bar{N}$ as before,

$$\Delta L_{fund} = \mathrm{Tr}\left(-\frac{1}{2}\left(\frac{\mu}{3}\right)^2\bar{q}_2q_2-\frac{1}{2}\left(\frac{\mu}{6}\right)^2\bar{q}_1q_1-\frac{1}{2}\mu X_5\,\bar{q}_2q_2-\frac{\mu}{4}\psi^\dagger\gamma_5\psi\right). \tag{2.39}$$

The susy transformation extends to

$$\begin{aligned}\delta\bar{q}_i &= -2i\psi^\dagger\epsilon_i(t)\,,\\ \delta\psi &= -D_0q_i\epsilon_i+i\sum_{\mu=1}^{5}q_iX_\mu\gamma_\mu\epsilon_i+\frac{i\mu}{6}\gamma_5q_1\epsilon_1+\frac{i\mu}{3}\gamma_5q_2\epsilon_2\,.\end{aligned} \tag{2.40}$$

As in the above adjoint hypermultiplet, the vacuum energy cancels among each hypermultiplet

$$N\times\left(2\times\frac{|\mu|}{3}+2\times\frac{|\mu|}{6}-4\times\frac{|\mu|}{4}\right)=0\,. \tag{2.41}$$

3. Deformed $\mathcal{N}=8$ superalgebra

Let us start by rewriting deformed $\mathcal{N}=16$ superalgebra of BMN in a form suitable for the Dirac matrices of section 2.2. Since the Dirac matrix is now complex, fermions and supercharges are all necessarily complex. The reality property is achieved by imposing a Majorana condition $\Psi=B\Psi^*$. Thus all 16 supercharge must satisfy,

$$\mathcal{Q}=B(\mathcal{Q}^\dagger)^T\,, \tag{3.1}$$

where the transposition is for the spinor indices only. To be definite, let us write their explicit form,[2]

$$\begin{aligned}\mathcal{Q}_M &= \left(P_I\bar{\gamma}_I+\frac{i}{2}[X_I,X_J]\bar{\gamma}_{IJ}-\frac{\mu}{3}X_a\bar{\gamma}_a\bar{\gamma}_{567}-\frac{\mu}{6}X_s\bar{\gamma}_s\bar{\gamma}_{567}\right)_{MK}\Psi_K\,,\\ \mathcal{Q}^\dagger_K &= \Psi^\dagger_M\left(P_I\bar{\gamma}_I-\frac{i}{2}[X_I,X_J]\bar{\gamma}_{IJ}+\frac{\mu}{3}X_a\bar{\gamma}_a\bar{\gamma}_{567}-\frac{\mu}{6}X_s\bar{\gamma}_s\bar{\gamma}_{567}\right)_{MK}\,,\end{aligned} \tag{3.2}$$

[2] $\bar{\gamma}_I$'s in this section differ from those introduced in section 2.1. Instead, we have $\bar{\gamma}_I=\Gamma^0\Gamma_I$ in terms of complex Dirac matrices of (2.13).

with SO(9) spinor indices M and K. As usual the canonical commutators are $[P_I, X^J] = -i\delta_I^J$. The anticommutators of the supercharges are[3]

$$\{\mathcal{Q}_M, \mathcal{Q}_K^\dagger\} = \delta_{MK} 2\mathcal{H} - \frac{\mu}{3}(\bar{\gamma}_{ab}\bar{\gamma}_{567})_{MK}\,\mathcal{L}_{ab} + \frac{\mu}{6}(\bar{\gamma}_{st}\bar{\gamma}_{567})_{MK}\,\mathcal{L}_{st} \\ + i(\bar{\gamma}^I)_{MK}\,\mathrm{Tr}\left[X^I \mathcal{G}\right], \tag{3.4}$$

where $\mathcal{G}$ is the Gauss constraint. As before, a, b run over 5,6,7, while s, t run over 1,2,3,4,8,9. Supercharges do not commute with hamiltonian in these mass deformed theories, but raise or lower energy as follows,

$$[\mathcal{H}, \mathcal{Q}_K^\dagger] = -i\frac{\mu}{12}\mathcal{Q}_M^\dagger\,(\bar{\gamma}_{567})_{MK}\,. \tag{3.5}$$

As a matter of convenience, we regard this theory as $\mathcal{N} = 8$ with a single adjoint hypermultiplet, and consider a subset of 8 supercharges. Recall that

$$\Gamma_{11} = 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3\,, \tag{3.6}$$

with respect to which all fermions and thus all 16 supercharges must be of $+1$ eigenvalue. Thus 16 supercharges are split into two classes;

$$\mathcal{Q} = Q \oplus Q'\,, \tag{3.7}$$

where Q_α^i are $(+,+)$ eigenvalues with respect to two σ_3 factors of Γ_{11} and $(Q')_\alpha^i$ are of $(-,-)$ eigenvalues. The reduction we took involves keeping the former.

Let us isolate part of algebra relevant to truncation to $\mathcal{N} = 8$. First of all, we introduce different notations for the hamiltonian and the angular momenta,

$$\mathcal{H} \to H, \qquad \mathcal{L} \to L \tag{3.8}$$

respectively, to emphasize that we are now considering $\mathcal{N} = 8$ theories. While we write down the algebra for the case of $\mathcal{N} = 8$ theory with a single adjoint hyper, generalization to more hypermultiplets is straightforward to modifying the hamiltonian H and the angular momentum L.

Reduction to $\mathcal{N} = 8$ is a matter of picking out $(+,+)$ eigensectors under Γ_{11}. This means that of various combinations of Dirac matrices appearing in the right hand side of eq. (3.4), only those that are diagonal under the decomposition $\mathcal{Q} = Q \oplus Q'$ may survive the truncation. We list all such combinations, and their truncated form as $(4\times 4)\otimes(2\times 2)$,

$$\begin{aligned} \bar{\gamma}_{67}\bar{\gamma}_{567} &\to \gamma_5 \otimes (-1)\,, \\ \bar{\gamma}_{89}\bar{\gamma}_{567} &\to \gamma_5 \otimes (+1)\,, \\ \bar{\gamma}_{\mu\nu}\bar{\gamma}_{567} &\to \gamma_{\mu\nu}\gamma_5 \otimes i\sigma_3 \qquad (\mu,\nu = 1,2,3,4)\,, \\ \bar{\gamma}_\mu &\to \gamma_\mu \otimes \sigma_3 \qquad (\mu = 1,2,3,4,5)\,, \end{aligned} \tag{3.9}$$

[3]Our convention is such that

$$\mathcal{L}_{IJ} = X_I P_J - X_J P_I + \cdots. \tag{3.3}$$

in order of their appearance in (3.4). The reduced superalgebra is then,

$$\{Q_{\alpha i}, Q^\dagger_{\beta j}\} = \delta_{ij}\delta_{\alpha\beta}\, 2H + \mu\, \delta_{ij}(\gamma^5)_{\alpha\beta}\,[\,T\,] + \frac{\mu}{3}\,\delta_{ij}(\gamma^5)_{\alpha\beta}\,[\,S\,] + \\ + \frac{\mu}{6}\sum_{\mu,\nu=1}^{4} i(\sigma_3)_{ij}(\gamma^{\mu\nu}\gamma^5)_{\alpha\beta}\,[L_{\mu\nu}] + \sum_{\mu=1}^{5} i(\sigma_3)_{ij}(\gamma^\mu)_{\alpha\beta}\,\mathrm{Tr}\,[X^\mu \mathcal{G}]\,, \qquad (3.10)$$

and

$$[H, Q^\dagger_{\beta j}] = \frac{\mu}{12} Q^\dagger_{\alpha i}(\gamma^5)_{\alpha\beta}(\sigma_3)_{ij}\,, \qquad (3.11)$$

with

$$S \equiv \frac{1}{2}\,[L_{67} - L_{89}]\,, \\ T \equiv \frac{1}{2}\,[L_{67} + L_{89}]\,. \qquad (3.12)$$

See below for more detailed discussion of angular momenta and how they extends when we includes more hypermultiplets.

$L_{\mu\nu}$ $(\mu,\nu = 1,2,3,4)$ and S constitute the generators of the R-symmetry $\mathrm{SO}(4)_{1,2,3,4} \times \mathrm{SO}(2)_{67-89}$. The other generator T, on the other hand, does not correspond to an R-symmetry in $\mathcal{N} = 8$ language. Instead it rotates the entire hypermultiplet as a whole by a U(1) phase. In particular T will drop away if we remove all hypermultiplets. We summarize below how various fields are charged under (S, T),

		S	T
X_μ	$\to$	0	0
$\lambda_1 = \lambda$	$\to$	$-1/2$	0
$\lambda_2 = -i\tilde{B}\lambda^*$	$\to$	$+1/2$	0
y_1	$\to$	$+1/2$	$-1/2$
y_2	$\to$	$-1/2$	$-1/2$
χ	$\to$	0	$-1/2$

(3.13)

For more general theories with other hypermultiplets the same algebra hold as long as we properly extend the operators to include these additional matter fields. Two SO(2) global charges of the general hypermultiplets follows those of adjoint hypermultiplet;

		S	T
q_1	$\to$	$+1/2$	$-1/2$
q_2	$\to$	$-1/2$	$-1/2$
ψ	$\to$	0	$-1/2$

(3.14)

Finally $L_{\mu\nu}$ with $\mu, \nu = 1, 2, 3, 4$ which rotates $X^{1,2,3,4}$ acts on all fermions universally via $\Gamma_{\mu\nu} = \gamma_{\mu\nu} \otimes 1 \otimes 1 \otimes 1$.

Actually, there is only one independent, 4-component supercharge $Q_\alpha \equiv Q^1_\alpha$, since $Q^2 = -i\tilde{B}((Q^1)^\dagger)^T$. For the sake of completeness, we write down an explicit form of $Q^\dagger$

with one adjoint hypermultiplet. Introducing new notation π_i for the conjugate momenta of $\bar{y}_i$'s such that $[\pi_i, \bar{y}_j] = -2i\delta_{ij}$, we have

$$
\begin{aligned}
Q^\dagger_\alpha = \lambda_1^\dagger &\left(\gamma_\mu P_\mu - \frac{i}{2}[X_\mu, X_\nu]\gamma^{\mu\nu} + i\frac{\mu}{3}X^5 - i\frac{\mu}{6}\sum_{\mu=1}^{4} X_\mu \gamma^\mu \gamma^5\right) + \\
&+ i\lambda_1^\dagger \left(\frac{[\bar{y}_2, y_2] - [\bar{y}_1, y_1]}{2}\right) - i\lambda_2^\dagger [\bar{y}_2, y_1] + \\
&+ \chi_1^\dagger \left(-\pi_1 - i[X_\mu, y_1]\gamma^\mu + i\frac{\mu}{6} y_1 \gamma^5\right) + \\
&+ \chi_2^\dagger \left(-\bar{\pi}_2 - i[X_\mu, \bar{y}_2]\gamma^\mu - i\frac{\mu}{3}\bar{y}_2 \gamma^5\right) .
\end{aligned} \tag{3.15}
$$

One must keep in mind that

$$
\begin{aligned}
\lambda_1 &= \lambda\,, \qquad & \lambda_2 &= -i\tilde{B}\lambda^*\,, \\
\chi_1 &= \chi\,, \qquad & \chi_2 &= +i\tilde{B}\chi^*\,.
\end{aligned} \tag{3.16}
$$

Adding more hypermultiplets is done straightforwardly by copying the hypermultiplet part (last 3 lines) and changing the gauge indices. Then, the above superalgebra may be rewritten without the symplectic indices as

$$
\begin{aligned}
\{Q_\alpha, Q_\beta\} &= 0\,, \\
\{Q_\alpha, Q^\dagger_\beta\} &= \delta_{\alpha\beta} 2H + \mu(\gamma_5)_{\alpha\beta}\,[\,T\,] + \frac{\mu}{3}(\gamma_5)_{\alpha\beta}\,[\,S\,] + \frac{\mu}{6}\sum_{\mu,\nu=1}^{4} i(\gamma^{\mu\nu}\gamma^5)_{\alpha\beta}\,[L_{\mu\nu}] + \\
&\quad + \sum_{\mu=1}^{5} i(\gamma^\mu)_{\alpha\beta}\ \mathrm{Tr}\,[X^\mu \mathcal{G}]\,,
\end{aligned} \tag{3.17}
$$

and

$$
[H, Q^\dagger_\beta] = \frac{\mu}{12} Q^\dagger_\alpha (\gamma^5)_{\alpha\beta}\,. \tag{3.18}
$$

4. DLCQ of fivebranes in PP-wave background and giant gravitons

In ordinary M(atrix) theory, k longitudinal fivebranes can be studied by considering $\mathcal{N} = 8$ SYQM with k fundamental hypermultiplets. As is well known, this comes about because longitudinal fivebranes are actually D4 branes whereby D0-D4 strings are introduced. The fivebrane is one of more difficult objects to understand, largely because their mutual interaction is governed by tensorial theory instead of usual gauge theories.

In fact, even some of more elementary aspect of fivebrane dynamics are still mysterious, such as its number of degrees of freedom. The latter has been estimated to scale as k^3 for large k, both from study of holography [15] and from study of conformal [13] and axial [14] anomalies. No compelling, microscopic explanation of this counting is available at the moment. One of more concrete proposal for study of fivebrane is via DLCQ approach, which really boils down to study of the above mentioned M(atrix) theory with fundamental hypermultiplets. Here again, even some of more elementary questions prove very difficult to answer. For instance, one may ask quantum vacuum structure of such SYQM, but even this has not been answered thoroughly. This was answered in some special cases in refs. [16]–[20].

One may hope that simplification from the above mass deformation will give us some extra handle on fivebranes, in much the same way that we are hoping to learn more about string theory by studying its behavior in the pp-wave backgrounds, such as the vacuum structure. With k fivebranes lying along $x^{0,6,7,8,9,10}$, in addition to the fivebranes that gave rise to the pp-wave background, dynamics of N D0 partons are described by the above deformed $\mathcal{N}=8$ SYQM with one adjoint hypermultiplet and k fundamental hypermultiplets. In this we wish to explore supersymmetric vacua of such a theory.

4.1 Fuzzy spheres as giant gravitons in the bulk

Most intriguing objects found to date in the mass deformed $\mathcal{N}=16$ M(atrix) theory of BMN are the fuzzy sphere solutions. This consists of three nontrivial matrix X^a with $a=5,6,7$ such that

$$[X^a, X^b] = -\frac{i\mu}{3}\epsilon_{abc}X^c\,, \tag{4.1}$$

is satisfied [1]. This can be rescaled to the standard su(2) algebra. Remarkably, such nontrivial solutions are actually of zero energy and preserves all dynamical supersymmetries.

The solutions are classified into irreducible representations of su(2), so, for SU(N) SYQM, there are as many such solution as the number of inequivalent partitions of N. These classical solutions are strongly reminiscent of D0 bound states of original M(atrix) theory [21]. Indeed, the bound states are supposed to exist for any number of D0's, which means that, for SU(N) SYQM and $N=\sum_p N_p$, there are sectors with groups of N_p D0 particles bound together. In the current mass deformed version, translational degrees of freedom are lost, and everything is confined near the origin. Nevertheless it is tantalizing that hint of these D0 bound states still survives in the form of giant gravitons [12] trapped by the confining potential.

Recall that deformed matrix theory of BMN admits a set of classical vacua that preserve all dynamical supersymmetries. These zero energy solutions can be understood easily from the form of potential with $X^i=0$ for $i=1,2,3,4,8,9$.

$$V \to \mathrm{Tr}\left(-\frac{1}{4}\sum_{a,b=5,6,7}[X^a,X^b]^2+\frac{1}{2}\left(\frac{\mu}{3}\right)^2\sum_{a=5,6,7}(X^a)^2 - i\frac{\mu}{3}\epsilon_{abc}X^aX^bX^c\right), \tag{4.2}$$

which can be made into a complete square,

$$-\frac{1}{4}\sum \mathrm{Tr}\left([X^a,X^b]+\frac{i\mu}{3}\epsilon_{abc}X^c\right)^2. \tag{4.3}$$

A fuzzy sphere,

$$X^a = -\frac{\mu}{3}J^a\,, \tag{4.4}$$

with any N dimensional representation J^a of su(2), is a classical ground state.

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In fact this is sufficient to show that these are invariant under all dynamical supercharges once we note that superalgebra are of the form

$$\{Q_\alpha, Q_\beta\} = \delta_{\alpha\beta} 2H + \text{angular momentum}\,, \tag{4.5}$$

which, for purely bosonic configurations with zero momentum, reduces to

$$\{Q_\alpha, Q_\beta\} = \delta_{\alpha\beta} 2V\,. \tag{4.6}$$

Thus a purely bosonic solution with $V = 0$ and no momentum preserves all dynamical supersymmetries.[4]

Given U(N) matrix theory, there is precisely one such distinct state for each partition $N = \sum_p N_p$, corresponding to the direct sum of N_p dimensional irreducible representations of su(2). These classical states and its quantum counterpart is closest thing to a usual Kaluza-Klein modes of supergraviton, or equivalently BPS bound states of D-particles. In fact, one may think each of these $N_p \times N_p$ blocks as the bound state N_p D-particles that are blown-up into a spherical membrane due to Myers' dielectric effect [23].

4.2 BPS equations for $\mathcal{N} = 8$

It thus becomes of some importance to search for such supersymmetric classical and quantum vacua in the deformed matrix quantum mechanics with $\mathcal{N} = 8$ supercharges. In the simplest case of pure $\mathcal{N} = 8$ quantum mechanics with no hypermultiplet at all, adjoint or not, since Myers' cubic term is absent in this case, and rest of bosonic potential are positive definite, we can immediately see that no analogous classical solution exists. With one adjoint hypermultiplet, we basically come back to BMN case, where the only difference now is that instead of $X^{a=5,6,7}$, we use the notation $X^5 \to X^5$ and $X^6 + iX^7 \to \bar{y}_2$;

$$-\frac{1}{4}\,\mathrm{Tr}\left([X^a, X^b] + \frac{i\mu}{3}\epsilon_{abc}X^c\right)^2 \tag{4.7}$$

$$\to\ +\frac{1}{2}\,\mathrm{Tr}\left(\frac{1}{2}[\bar{y}_2, y_2] + \frac{\mu}{3}X^5\right)^2 - \frac{1}{2}\,\mathrm{Tr}\left(\left([X^5, \bar{y}_2] + \frac{\mu}{3}\bar{y}_2\right)\left([X^5, y_2] - \frac{\mu}{3}y_2\right)\right). \tag{4.8}$$

It is instructive to keep y_1 as well in the potential and write it in complete squares,

$$\begin{aligned}&+\frac{1}{2}\,\mathrm{Tr}\left(\frac{1}{2}[\bar{y}_2, y_2] - \frac{1}{2}[\bar{y}_1, y_1] + \frac{\mu}{3}X^5\right)^2 - \frac{1}{2}\,\mathrm{Tr}\left(\left([X^5, \bar{y}_2] + \frac{\mu}{3}\bar{y}_2\right)\left([X^5, y_2] - \frac{\mu}{3}y_2\right)\right) - \\ &-\frac{1}{2}\,\mathrm{Tr}\left(\left([X^5, \bar{y}_1] + \frac{\mu}{6}\bar{y}_1\right)\left([X^5, y_1] - \frac{\mu}{6}y_1\right)\right) + \frac{1}{8}\,\mathrm{Tr}\left([\bar{y}_1, y_2][\bar{y}_2, y_1]\right),\end{aligned} \tag{4.9}$$

which is more suitable for generalization.[5]

[4] More general class of solutions that preserves part of the supersymmetries may be found [22].

[5] While this may suggest a new class of ground states where $y_2 = 0$ instead of $y_1 = 0$, nontrivial y_1's which are zero of the potential does not satisfy hermiticity conditions. No such additional ground state exists.

Generalization to cases with additional hypermultiplets is obvious. Denoting complex scalars $q_i^{(f)}$ of the f-th fundamental hypermultiplets, let us write the reduced potential similarly

$$
\begin{aligned}
&+\frac{1}{2}\,\mathrm{Tr}\left(\frac{1}{2}[\bar{y}_2,y_2]+\frac{1}{2}\sum_f \bar{q}_2^{(f)}q_2^{(f)}-\frac{1}{2}[\bar{y}_1,y_1]-\frac{1}{2}\sum_f \bar{q}_1^{(f)}q_1^{(f)}+\frac{\mu}{3}X^5\right)^2\\
&-\frac{1}{2}\,\mathrm{Tr}\left(\left([X^5,\bar{y}_2]+\frac{\mu}{3}\bar{y}_2\right)\left([X^5,y_2]-\frac{\mu}{3}y_2\right)\right)\\
&-\frac{1}{2}\,\mathrm{Tr}\left(\left([X^5,\bar{y}_1]+\frac{\mu}{6}\bar{y}_1\right)\left([X^5,y_1]-\frac{\mu}{6}y_1\right)\right)\\
&+\frac{1}{2}\sum_f \mathrm{Tr}\left(\left(X^5\bar{q}_2^{(f)}+\frac{\mu}{3}\bar{q}_2^{(f)}\right)\left(q_2^{(f)}X^5+\frac{\mu}{3}q_2^{(f)}\right)\right)\\
&+\frac{1}{2}\sum_f \mathrm{Tr}\left(\left(X^5\bar{q}_1^{(f)}+\frac{\mu}{6}\bar{q}_1^{(f)}\right)\left(q_1^{(f)}X^5+\frac{\mu}{6}q_1^{(f)}\right)\right)\\
&+\frac{1}{8}\,\mathrm{Tr}\left(\left([\bar{y}_1,y_2]+\sum_f \bar{q}_1^{(f)}q_2^{(f)}\right)\left([\bar{y}_2,y_1]+\sum_f \bar{q}_2^{(f)}q_1^{(f)}\right)\right).
\end{aligned} \tag{4.10}
$$

In the presence of fundamental hypermultiplets, the equation for static classical vacua is then

$$
\begin{aligned}
-\frac{\mu}{3}X^5 &= \frac{1}{2}[\bar{y}_2,y_2]+\frac{1}{2}\sum_f \bar{q}_2^{(f)}q_2^{(f)}-\frac{1}{2}[\bar{y}_1,y_1]-\frac{1}{2}\sum_f \bar{q}_1^{(f)}q_1^{(f)}\,, && (4.11)\\
[X^5,\bar{y}_1] &= -\frac{\mu}{6}\bar{y}_1\,, && (4.12)\\
[X^5,\bar{y}_2] &= -\frac{\mu}{3}\bar{y}_2\,, && (4.13)\\
X^5\bar{q}_1^{(f)} &= -\frac{\mu}{6}\bar{q}_1^{(f)}\,, && (4.14)\\
X^5\bar{q}_2^{(f)} &= -\frac{\mu}{3}\bar{q}_2^{(f)}\,, && (4.15)\\
0 &= [\bar{y}_1,y_2]+\sum_f \bar{q}_1^{(f)}q_2^{(f)}\,. && (4.16)
\end{aligned}
$$

These conditions can also be derived from supersymmetry conditions: these are precisely condition for a purely bosonic solution to preserve all 8 dynamical supersymmetries. Eq. (4.11) and eq. (4.16) follow from $\delta\lambda = 0$ upon using equation of motion for the auxiliary fields, $\mathbf{D}$, while middle ones follow from $\delta\chi = 0$ and $\delta\psi = 0$ respectively.

4.3 Giant gravitons on fivebranes

Nontrivial solutions with $q = 0$ are of course the fuzzy sphere solutions again, and corresponds to states in the "bulk". In the presence of fivebranes, new supersymmetric solutions arise with $q \neq 0$. A particularly simple set of solutions can be found in abelian case, corresponding to a single D0 probing k fivebranes at origin in the pp-wave limit. X^5 could be either $-\mu/3$ for $q_1 = 0$ or $-\mu/6$ for $q_2 = 0$. However, nonnegativity of $\bar{q}_i q_i$ disallow the latter solution, so we must take $q_1 = 0$ and $X^5 = -\mu/3$. Commutators vanish identically, implying that $y_i = 0$ also, and the equations simplify to

$$\frac{1}{2}\sum_f |q_2^{(f)}|^2 = -\frac{\mu}{3}X^5 = \left(\frac{\mu}{3}\right)^2 . \tag{4.17}$$

Thus, a continuous set of solutions exist and are parametrized as

$$q_2^{(f)} = \sqrt{2}\,\frac{\mu}{3} z^f \,, \tag{4.18}$$

where z is any complex k-vector of unit length. Furthermore, we must identify vacua up to U(1) gauge rotation, and so the vacuum moduli space is CP^{k-1}. Quantization of this will lead to a unique supersymmetric ground state, since the low energy dynamics is purely bosonic, and should be related to the bound states found in ref. [18].

For nonabelian theories again with k hypermultiplet, there is a direct generalization of the above family of solutions. Take an $N \times N$ block of adjoint scalars and write

$$-(X^5)_{AB} = \frac{\mu}{3}(n+1-A)\delta_{AB} \,, \tag{4.19}$$

where $(N-1)/2 < n \leq N$ so that $(X^5)_{nn} = -\mu/3$. With $q_1 = 0 = y_1$, the following form of $\bar{q}_2$ and $\bar{y}_2$ solve the BPS equations

$$(\bar{q}_2^{(f)})_A = \sqrt{2}\,\frac{\mu}{3} w_2^f \delta_{A,n}, \qquad (\bar{y}_2)_{AB} = \sqrt{2}\,\frac{\mu}{3}\alpha_A \delta_{A,B-1} \tag{4.20}$$

with

$$\begin{aligned} |\alpha_1|^2 &= n \,, \\ |\alpha_2|^2 - |\alpha_1|^2 &= n-1 \,, \\ &\vdots \\ |\alpha_{n-1}|^2 - |\alpha_{n-2}|^2 &= 2 \,, \\ \sum |w_2^f|^2 + |\alpha_n|^2 - |\alpha_{n-1}|^2 &= 1 \,, \\ |\alpha_{n+1}|^2 - |\alpha_n|^2 &= 0 \,, \\ &\vdots \\ |\alpha_{N-1}|^2 - |\alpha_{N-2}|^2 &= n+2-N \,, \\ -|\alpha_{N-1}|^2 &= n+1-N \,. \end{aligned} \tag{4.21}$$

Note that this family of solution is again parametrized by CP^{k-1}.

Another set of solutions are

$$-(X^5)_{AB} = \frac{\mu}{3}\left(m + \frac{1}{2} - A\right)\delta_{AB} \,, \tag{4.22}$$

where $1 \leq m < N/2$ so that $(X^5)_{mm} = -\mu/6$. With $q_2 = 0 = y_1$, the following form of $\bar{q}_1$ and $\bar{y}_2$ solves the BPS equation,

$$(\bar{q}_1^{(f)})_A = \sqrt{2}\,\frac{\mu}{3} w_1^f \delta_{A,m} \,, \qquad (\bar{y}_2)_{AB} = \sqrt{2}\,\frac{\mu}{3}\alpha_A \delta_{A,B-1} \,, \tag{4.23}$$

with

$$
\begin{aligned}
|\alpha_1|^2 &= m - \frac{1}{2}\,, \\
|\alpha_2|^2 - |\alpha_1|^2 &= m - \frac{3}{2}\,, \\
&\vdots \\
|\alpha_{m-1}|^2 - |\alpha_{m-2}|^2 &= \frac{3}{2}\,, \\
-\sum |w_1^f|^2 + |\alpha_m|^2 - |\alpha_{m-1}|^2 &= \frac{1}{2}\,, \\
|\alpha_{m+1}|^2 - |\alpha_m|^2 &= -\frac{1}{2}\,, \\
&\vdots \\
|\alpha_{N-1}|^2 - |\alpha_{N-2}|^2 &= m + \frac{3}{2} - N\,, \\
-|\alpha_{N-1}|^2 &= m + \frac{1}{2} - N\,.
\end{aligned}
\tag{4.24}
$$

Each integer m ($1 \leq m < N/2$) or n ($(N-1)/2 < n \leq N$) gives distinct sets of solutions, which are each parametrized by CP^{k-1}.

5. Conclusion

In this note, we derived massive deformation of $\mathcal{N} = 8$ supersymmetric Yang-Mills quantum mechanics, analogous to BMN's deformation of M(atrix) theory. A particular case with one adjoint hypermultiplet and k fundamental hypermultiplet is interpreted as D0 dynamics in the presence of k longitudinal fivebranes, in the M-theory pp-wave background. A very rich structure of vacua is shown to exist when fundamental hypermultiplets are present, typically parametrized by CP^{k-1} vacuum moduli spaces.

Clearly these states are deformation of fuzzy spheres, or giant gravitons, of $\mathcal{N} = 16$ deformed M(atrix) theory of BMN, where deformation entails turning on fundamental hypermultiplets. It is tempting to view these states, since they protrude into the Higgs phase, as giant gravitons "trapped" by fivebranes. However, it remains largely mysterious what role they may play in dynamics of fivebranes.

Another aspect that needs be addressed is the matter of quantum ground states, as opposed to classical ground states. Proliferation of classical ground states will be in part lifted once quantum consideration is used; for each family of solutions parametrized by CP^{k-1} vacuum moduli space, one quantum ground state should exist and probably no more for instance. Assuming that no tunneling occurs between moduli spaces of different types of solution, such a ground state is likely to be fully supersymmetric and thus protected from correction. More detailed properties of quantum spectra of these $\mathcal{N} = 8$ theories will be investigated elsewhere.

Before closing, it is worthwhile to note that the kind of reduction process we have taken can be repeated for $\mathcal{N} = 8$ theory. For instance, start with the deformed $\mathcal{N} = 8$ pure SYQM of section 2.3, truncate away X^4, X^5 while restricting λ to be chiral under γ^5. At the end of the day, one obtains a pure $\mathcal{N}=4$ SYQM which is mass deformed by μ. Again for this theory, cancellation of vacuum energy is complete,

$$3\times\frac{\mu}{6}-2\times\frac{\mu}{4}=0\,, \tag{5.1}$$

since, in each vector multiplet, there are 3 bosons of mass $\mu/6$ and 2 complex fermions of mass $\mu/4$. At the moment it is unclear to us what is a pp-wave interpretation, and indeed whether there is one. Regardless of stringy interpretation, however, such large class of mass-deformed SYQM may provide further insights on issues on vacuum structure of general SYQM.

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